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Combustor Model for Simulation of Combustion Instabilities and Their Active Control

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The development and application of a numerical model of an actively controlled combustor are described. The main objective was to develop a model that could be used as a platform for studying closed-loop control of unstable combustors. The model uses a heuristic approach and global kinetics to describe mixing and combustion processes in a one-dimensional gaseous combustor. Initially, the model is used to investigate a combustor's response to open-loop excitation by periodic fuel injection. The numerical model's prediction that 1) the pressure oscillation amplitude decreases and 2) the phase shift between the fuel injection rate and heat-release oscillations increases linearly, as the frequency of fuel injection rate increases, are in good agreement with experimental data from a gas rocket. Next, closed-loop control is studied. The investigated controller determines the characteristics of the most unstable mode in real time and damps the instability by modulating the injection rate of a fraction of the injected fuel to excite a secondary combustion process within the combustor out of phase with respect to the most unstable mode of pressure oscillations. Active control examples include the study of the dependence of the controller's performance on the amount of fuel pulsed.

I. Introduction

THE occurrence of combustion instabilities often hinders the development of propulsion systems. It could cause a catastrophic failure of the whole system (e.g., in a rocket motor), or reduce performance (e.g., in a lean premixed combustor) by forcing it to operate at an equivalence ratio significantly higher than the lean flammability limit, thereby increasing the NO_x emissions. Recently, active control has emerged as a highly promising approach for controlling combustion instabilities. The design of an active-control system requires, however, the use of models that can simulate the behavior of the controlled system. The development of such a model for a controlled combustor and its performance are the subject of this paper.

Several approaches have been investigated for active control of combustion instabilities.[1] Active control of instabilities is, however, a relatively new field and the development of active controllers for practical combustors demands further efforts. For example, in a study of adaptive active control of combustion instabilities,[2] it was observed that under certain operating conditions the control action can destabilize some natural acoustic modes of the system. Another study[3] discusses problems related to the adaptive filter becoming unstable in the process of controlling combustion instabilities. Such problems could be eliminated in the design stage if a model capable of simulating the performance of the actively controlled system was available and used to guide the design of the actively controlled combustor.

The performance of controlled combustors can be investigated by reducing the governing equations to a system of ordinary differential equations using, for example, the Galerkin method, or by solving them numerically. The latter approach generally requires considerable computational effort and is more suitable for developing an understanding of the fundamental processes that govern the controlled combustor's performance. For example, the numerical simulations of Menon[4,5] focus on the detailed interactions between the unsteady heat release and vortical and acoustic wave motions with little emphasis on the performance of the active controller. Such numerical simulations are capable of predicting the domains of instability for a specific combustor but are not likely to be used as a design tool due to their high computational cost. Consequently, there exists a need for developing a simplified controlled combustor model whose solution will be computationally efficient and could be used to investigate the performance of controlled combustors.

The development and performance of such a model are described in the present study. The model developed uses a heuristic approach and global kinetics to describe mixing and reaction processes, respectively. It was developed to study active control of axial instabilities in a constant-area combustor that burns premixed, gaseous reactants. Comparisons were made with data from a gas rocket. To apply the present model to other types of combustors, modifications involving, for example, the characterization of the spatial dependence of mixing, are required. Active control is attained by modulating the injection rate of a secondary fuel stream that damps the instability by producing secondary combustion and heat-addition processes within the combustor out of phase with respect to the unstable combustor-pressure oscillations.

II. Heuristic Combustor Model

This section describes the derivation of a simplified model that can be used to qualitatively study the performance of an actively controlled combustor with relatively little computational cost. Special emphasis is placed on developing a model that could account for the effects of mixing, convection, and heat-addition processes on the onset of one-dimensional instabilities within the combustor and the effect of various control strategies on the combustor's stability. The derivation starts with the following Euler equations:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}}{\partial x} = \boldsymbol{S} \tag{1}$$

where $\boldsymbol{U} = \{\rho_f, \rho_{ox}, \rho, \rho u, e\}^T$, with ρ_f, ρ_{ox}, and ρ representing the densities of fuel, oxygen, and mixture, respectively. $\boldsymbol{F} = \{\rho_f u, \rho_{ox} u, \rho u, \rho u^2 + p, u(e + p)\}^T$ and e is the total energy per unit volume. The source term $\boldsymbol{S}$ that represents the effects of

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chemical reaction and injection of secondary fuel into the combustor in a direction perpendicular to the main flow is given by

$$S = \{\omega_f + \varepsilon_f, \omega_{ox}, \varepsilon_f, 0, H_s\varepsilon_f - \omega_f Q_{cal}\}^T \quad (2)$$

where ω_f and ω_{ox} are the reaction rates of the fuel and oxygen (related through stoichiometry), ε_f is the secondary fuel addition rate per unit volume, H_s is the stagnation enthalpy of the secondary fuel, and Q_{cal} is the calorific value of the fuel.

Next, Eq. (1) is modified by the addition of another source term that heuristically describes the mixing of cold reactants with hot gases by large-scale fluid motions. It is proposed to start with a source term of the form $W(x)(\boldsymbol{U}_{mix} - \boldsymbol{U})/\tau_{mix}$, where $\boldsymbol{U}_{mix}$ is a yet to be defined average value of $\boldsymbol{U}$ in the mixing region, τ_{mix} is a prespecified characteristic mixing time that determines the rate of the mixing process, and $W(x)$ is a prespecified weighting function that describes the spatial dependence of the mixing process. Adding the heuristic mixing term to Eq. (1) yields

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}}{\partial x} = W(x)\frac{(\boldsymbol{U}_{mix} - \boldsymbol{U})}{\tau_{mix}} + \boldsymbol{S} \quad (3)$$

The mixing term should act only as a redistributive term and not serve as an additional source of mass, momentum, or energy. Consequently, after integrating Eq. (3) over the total combustor volume, the integral corresponding to the contribution of the mixing term should go to zero. This constraint yields the following definition of $\boldsymbol{U}_{mix}$:

$$\boldsymbol{U}_{mix}(t) = \frac{\int_0^L W(x)\boldsymbol{U}(x,t)\,dx}{\int_0^L W(x)\,dx} \quad (4)$$

The form of the selected source term $W(x)(\boldsymbol{U}_{mix} - \boldsymbol{U})/\tau_{mix}$ is similar to that of another mixing model,[6] known as the interaction by exchange with the mean (IEM) model that was originally used to investigate chemical engineering problems. Some aspects of the IEM model that are different from our mixing model are as follows: 1) the IEM model is a Lagrangian model, 2) the IEM model does not involve any weighting function $W(x)$, and 3) the average concentration for a species is obtained from a residence time distribution.

The heuristic mixing source term in Eq. (3) contains only one timescale parameter, namely, τ_{mix}. The need for the introduction of another timescale parameter in the heuristic mixing source term is discussed next.

Because our model is oriented toward combustion instability control applications, the main interest is in capturing the overall behavior of the combustor as a dynamical system. Previous studies[7–9] that describe the effect of the flame in terms of a transfer function involve the use of a pure time delay for characterizing the combustor dynamics. This emphasizes the need for the introduction of pure time delay parameter(s) in the heuristic combustor model. Physical processes such as mixing and convection are characterized by pure time delays. The introduction of a pure time delay type of behavior associated with mixing, through a modification of the heuristic mixing source term, is discussed first. Accounting for a pure time delay associated with convection will be discussed later.

To incorporate a pure time delay due to mixing in our model, we start by noting that the unsteady term $\partial\boldsymbol{U}/\partial t$ depends on $(\boldsymbol{U}_{mix} - \boldsymbol{U})/\tau_{mix}$, which acts as a driving term. This dependence is analogous to that exhibited by the first-order ordinary differential equation $dy/dt = (x - y)/\tau_{mix}$ (where x and y are the input and output, respectively) whose transfer function is $1/(1 + \tau_{mix}s)$. Thus, the effect of the mixing source term is to drive the local property $\boldsymbol{U}$ toward an average ($\boldsymbol{U}_{mix}$) at a mixing rate controlled by the characteristic time τ_{mix}. In the frequency domain, the effect of a pure time delay associated with a mixing process, denoted as $\tau_{d,mix}$, can be included in $1/(1 + \tau_{mix}s)$ by the factor $e^{-\tau_{d,mix}s}$, which is the transfer function for a pure time delay $\tau_{d,mix}$, to yield $e^{-\tau_{d,mix}s}/(1 + \tau_{mix}s)$. In the time domain, this suggests replacing $(\boldsymbol{U}_{mix} - \boldsymbol{U})/\tau_{mix}$ with $(\boldsymbol{U}_{d,mix} - \boldsymbol{U})/\tau_{mix}$ where

$$\boldsymbol{U}_{d,mix}(t) = \boldsymbol{U}_{mix}(t - \tau_{d,mix}) \quad (5)$$

The governing equations are then given by

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}}{\partial x} = W(x)\frac{(\boldsymbol{U}_{d,mix} - \boldsymbol{U})}{\tau_{mix}} + \boldsymbol{S} \quad (6)$$

The physical significance of τ_{mix} and $W(x)$ in our model is discussed next. Equation (6) shows that the magnitude of the mixing term is proportional to $1/\tau_{mix}$. In the limiting case, when τ_{mix} approaches zero, the system is mathematically stable only if $\boldsymbol{U} = \boldsymbol{U}_{mix}$ in the region where $W(x)$ is not zero. That is, the properties are uniform in the mixing region. Thus, when τ_{mix} approaches zero, this mixing model resembles the well-stirred reactor model[10] that has been employed in the simplified steady-state analysis of reacting systems.

The weighting function $W(x)$ [see Eq. (6)] specifies the spatial dependence of the rate of the mixing process that generally depends on the combustor geometry and the reactants' injection pattern and velocities. For example, the rates of mixing processes in rocket motors that burn gaseous or liquid propellants are expected to depend on the characteristics of the regions where fuel and oxidizer jets impinge on one another and on the shear layers and recirculation regions that form between the injected reactants' jets where high rates of mixing occur. Taking a radial traverse of a rocket combustor flow near the injector plate, one would cross several shear layers and recirculation regions imbedded within the high-velocity reactants jets. Similar regions will be encountered if one traversed the combustor along the azimuthal direction at fixed radial and axial locations near the injector plate. Rapid mixing will generally occur at the interfaces of the recirculating regions and the reactants jets and at the interfaces where the reactants' jets impinge. These observations indicate that the mixing processes are highly three dimensional. The objective of the weighting function $W(x)$ is to heuristically account for the multidimensionality of the mixing process in the one-dimensional combustor model that is used in this study. Using intuition, we have assumed that $W(x)$ attains a finite maximum value at a short distance from the injector plate, where mixing processes are expected to dominate and that it decays exponentially with distance along the combustor as the reactants are consumed. This is clearly an approximation as the characteristics of $W(x)$ are expected to depend on the injection system design and undoubtedly vary from one rocket combustor to another.

In general, the function $W(x)$ should be chosen based on the combustor that is being modeled. For example, if the application of interest is a bluff body stabilized combustor, an appropriate choice for $W(x)$ would depend on the location, size, and shape of the bluff body flame holder because the spatial dependence of mixing is characterized by the recirculation zone and shear layers associated with the bluff body.

III. Investigated Combustor Geometry

The combustor model developed is applied to a gas rocket, facilitating comparison with data from a parallel experimental study.[11] A schematic of the combustor geometry investigated in this numerical study is shown in Fig. 1. In many aspects it emulates the experimental setup used in a parallel effort to develop an active-control

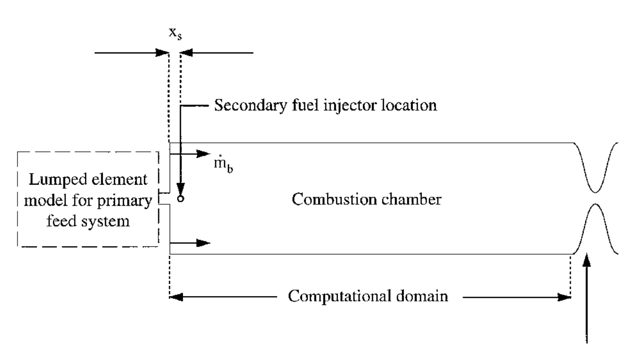


Fig. 1 Schematic of the investigated combustor.

system.[11] There is, however, a significant difference in the treatment of the primary reactant feed system in the experiment and in this numerical study. Whereas in the experimental rig the reactant influx was always choked, for some cases in this numerical study a coupling between the primary reactant flow rate and the combustor pressure has been assumed. This deviation was necessary for demonstration of combustion instabilities. The present authors believe that generation of combustion instabilities is possible with the developed model by coupling the parameters of the heuristic model to the pressure in the combustor even when the primary reactant flow is choked. However, in the present study it was decided to create a simpler instability mechanism on which an active-control system could be numerically tested. The assumed coupling involves a lumped element model for the primary feed system that creates a phase lag between the pressure oscillations in the injector interface and the mass flux [$\dot{m}_b(t)$] of premixed primary reactants into the combustor. This provides a simple means to account for the effect of the feed system on the combustor dynamics. In contrast to the primary inflow that may or may not be choked (depending on the numerical example), the orifice supplying the secondary (control) fuel into the combustor is always choked and, consequently, the secondary fuel flow rate is not affected by the combustor pressure oscillations. Thus, the control signal, which is the perturbation in secondary fuel injection, completely determines the amount of secondary fuel injected into the combustor at any time.

Because methane was used as the fuel in the parallel experimental gas rocket study,[11] it was also used in this numerical study and its reaction rate ω_f [see Eq. (2)] was modeled using a one-step global kinetic mechanism.[12]

The developed solutions were required to satisfy the following injector boundary conditions:

$$T_0 = \text{const}, \quad \rho u = \dot{m}_b / A \quad \text{at} \quad x = 0 \tag{7}$$

where T_0, $\dot{m}_b$, and A are the inlet stagnation temperature of the primary supply of reactants into the combustor, the corresponding mass flux, and the combustor cross sectional area, respectively. If the primary inflow into the combustor is choked, $\dot{m}_b$ is specified as a constant. This choked inlet boundary condition is used to compare the predictions of the numerical model with experimental data in the first numerical example in Sec. V. For a case with primary inflow that is not choked, a lumped element feed system model is used to determine the dependence of $\dot{m}_b$ on the combustor pressure because the elements of the feed system are outside the computational domain (see Fig. 1).

In the preceding section, the choice for the heuristic source term that models mixing was influenced, in part, by the need to account for a pure time delay associated with mixing processes. The dynamical behavior of some combustors has been characterized by a pure time delay associated with convection.[7,8] The need and means to account for a pure time delay associated with convection is discussed next for the injector boundary condition of the primary inflow.

In a rocket motor, the reactants pass through a complex flowfield before they react. The mixing and subsequent reaction processes introduce a time delay between the instants of injection and heat release that may be responsible for driving the instability.[13] Convection contributes to a significant fraction of this time delay. In fact, in an experimental study of instabilities in a gaseous rocket motor,[14] the injector design, which affects the convective and mixing processes, has been identified as the most important factor in influencing the occurrence of instabilities. For a given fluid element, consider the time delay between 1) the instant of injection into the combustion chamber and 2) the instant when the fluid element releases its heat content. This time delay, which depends on convection, is different for the experimental combustor and for our simplified one-dimensional combustor model. To alter this time delay in our combustor model, the boundary condition specifying the flow rate of reactants into the combustor can be shifted in time. This could be done by using $\dot{m}_b(t - \tau_{d,c})$ instead of $\dot{m}_b(t)$ in Eq. (7). A similar concept of pure time delay has been used by Feiler and Heidmann[15] to study combustion instabilities in rockets.

For the exit boundary condition, it is assumed that the nozzle at the downstream end of the combustion chamber is compact and choked. Using the quasi-steady approximation for the compact nozzle flow, we obtain

$$M = \text{const} \quad \text{at} \quad x = L \tag{8}$$

where M is the Mach number.

To employ the heuristic mixing model for studying this combustor, an expression for $W(x)$ should be chosen. Generally, the characteristics of the mixing processes in a rocket motor depend on the type of injector used. In this study, due to lack of data describing the characteristics of the mixing processes, the function $W(x)$, where x is the distance from the injector face, is described, for simplicity, by the expression (refer to the discussion at the end of Sec. II)

$$W(x) = \begin{cases} 0 & 0 < x < x_s \\ e^{-(x - x_s)/l_{\text{mix}}} & x_s \le x < L \end{cases} \tag{9}$$

where l_{mix} is a characteristic mixing length, L is the length of the combustor, and x_s is the location of the secondary fuel injection. In choosing the $W(x)$ given in Eq. (9), the primary concern is to model mixing in the reaction zone and its vicinity because this has a significant influence on the heat release and, hence, on the instability characteristics. Although mixing might occur between combustion products and excess air far downstream of the reaction region, $W(x)$ has been chosen to be of negligible magnitude in this region [see Eq. (9)] because the present study attempts to model mixing only in the region where it has a significant influence on the heat release dynamics of the system. The value of $W(x)$ is chosen to be zero for $x < x_s$ and a maximum at $x = x_s$ because, in the experimental gas rocket, the location of the secondary fuel injector was chosen to approximately coincide with the upstream end of the primary combustion region. This choice of the secondary fuel injector location minimizes the time delay between injection of secondary fuel and the resultant heat release. The location of the secondary fuel injector plays a significant role in the pure time delay that characterizes the combustion of the secondary fuel. For the $W(x)$ chosen in Eq. (9), the secondary fuel injector location x_s coincides with the start of the mixing zone. Because of this, the combustion of the secondary flow is predominantly influenced by the pure time delay $\tau_{d,\text{mix}}$. On the other hand, the combustion process associated with the primary reactants is not only influenced by $\tau_{d,\text{mix}}$, but also by the pure time delay due to the one-dimensional convection of the primary reactants from the injector face to the start of the mixing zone and the additional contribution due to $\tau_{d,c}$ if it is nonzero.

IV. Active Control System

The driving of the combustor-pressure oscillations by the heat-release oscillations is governed by Rayleigh's criterion.[16] It states that when the magnitude of the phase difference between the local pressure and heat-release oscillations is smaller or larger than 90 deg, the heat release oscillations tend to drive or damp the pressure oscillations, respectively. Rayleigh's criterion thus suggests that combustion instabilities can be controlled using the secondary (i.e., control) fuel injection to generate secondary heat release oscillations within the combustor 180 deg out of phase with respect to the unstable pressure oscillations. For effective operation of the active-control system, the location of the secondary fuel injection should be such that the oscillatory heat release produced by the secondary fuel injection should occur at or near a region of maximum pressure oscillations.

The active-control system investigated in this study is based on an observer[17] (this control approach has been tested in the parallel experimental effort[18]) that can track the most dominant modes of the instability in real time (see Fig. 2). This is essential because, in general, the nature of an instability (in terms of its constituent modes) in a practical system may change with time. A pressure signal from the combustor serves as the input to the observer that is set to determine the characteristics of the most dominant mode in real time. The control signal specifies the amount and phase of

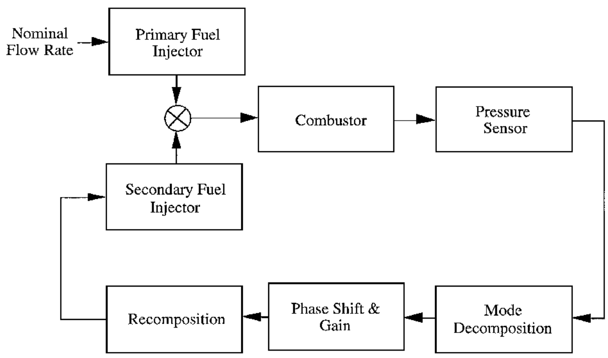


Fig. 2 Schematic of the investigated control system.

secondary fuel injection. The heat release produced by the secondary fuel injection damps the unstable pressure oscillations.

Next, we estimate the controller phase shift for our active-control system. Choosing the appropriate controller phase shift is crucial because it determines the phase difference between the secondary heat-release and unstable pressure oscillations, which, in turn, determines whether the secondary fuel injection will damp or amplify the combustor pressure oscillations. The temporal variation of the amount of secondary fuel injected into the combustor and, thus, its phase difference with the local pressure oscillations can be accurately controlled by using an appropriate actuator. The main issue in attempting to damp the instability is, however, to make sure that the heat release due to the secondary fuel is out of phase with the existing pressure oscillations. In our system, because the distance between the sensor and actuator is much smaller than the characteristic acoustic wavelength, if the heat release associated with the secondary fuel injection occurs instantaneously after the secondary fuel is injected, a controller phase shift of π to the dominant mode of the pressure signal will ensure that the dominant mode will be damped. However, the heat release is not instantaneous and is determined by, for example, mixing and chemical kinetic processes that are difficult to model. In our model, the phase difference between the secondary fuel injection and the corresponding heat release is mainly determined by the mixing term (refer to the first example in Sec. V for comparison with experimental data) and is given by the phase of $e^{-\tau_{d,\mathrm{mix}} s}/(1 + \tau_{\mathrm{mix}} s)$ (see the discussion in Sec. II) that is $-\omega\tau_{d,\mathrm{mix}} - \tan^{-1}(\omega\tau_{\mathrm{mix}})$ where ω is the instability frequency.

In a practical implementation of an active-control system, one also needs to account for the time delay due to devices that include an antialiasing filter and digital to analog and analog to digital converters. Denoting this time delay by τ_{con}, the corresponding phase shift is given by $-\omega\tau_{\mathrm{con}}$. Nelson and Elliott[19] provide an estimate for τ_{con} in active noise control systems.

By consideration of the cumulative effect of the preceding phase shifts, the required controller phase shift that will produce heat-release oscillations due to the secondary fuel out of phase with the existing pressure oscillations is chosen as $\pi + \omega\tau_{d,\mathrm{mix}} + \tan^{-1}(\omega\tau_{\mathrm{mix}}) + \omega\tau_{\mathrm{con}}$, with ω being the observer's current estimate of the instability frequency.

Note that our simplified approach of simulating the behavior of a combustor using a few model parameters has the additional advantage that some of the combustor model parameters ($\tau_{d,\mathrm{mix}}$ and τ_{mix} in this case) also serve as controller parameters for the active-control system. This can also be applicable for active control approaches other than the observer-based control system studied in the present work. For example, an adaptive control system requires information about the behavior of the physical system in the form of an auxiliary path transfer function.[2,3,19] The pure time delay $\tau_{d,\mathrm{mix}}$ can serve as an estimate for the auxiliary path transfer function (if it is modeled as just a pure time delay), or a part of the auxiliary path transfer function (if a delayed inverse model, one type of model for handling systems with pure time delay,[20] is employed).

Next we discuss the choice of the controller gain. A low controller gain, as expected, would result in inadequate damping of the instability. On the other hand, a high controller gain will cause the control system to be oversensitive to noise, especially when the controller maintains the pressure oscillations at a low level. To determine the controller's gain, the numerical simulation is run with the controller off and with model parameters that result in a large-amplitude instability. The controller's gain [units (kg/s)/Pa] is then chosen as the ratio of the oscillation amplitudes of the instantaneous total fuel consumption rate and the pressure at the injector face, which is a pressure antinode. In our simple approach, the controller gain is constant in time, whereas the controller phase shift is a function of time because it involves the instability frequency. Note that in the active control demonstration for the parallel experimental work,[18] both the controller gain and the phase shift are chosen based on the instability frequency from a lookup table and, hence, are functions of time. The lookup table for controller gain and phase shift was constructed as a function of frequency based on open-loop tests that involve pulsation of secondary fuel at different frequencies.[11]

The study of the active control of the combustion instabilities necessitates an understanding of the effect of various factors that influence the performance of the control system. As an example, this study employs the combustor model developed to investigate the dependence of the performance of the proposed control system on the flow rate of fuel used to control an instability of a given amplitude.

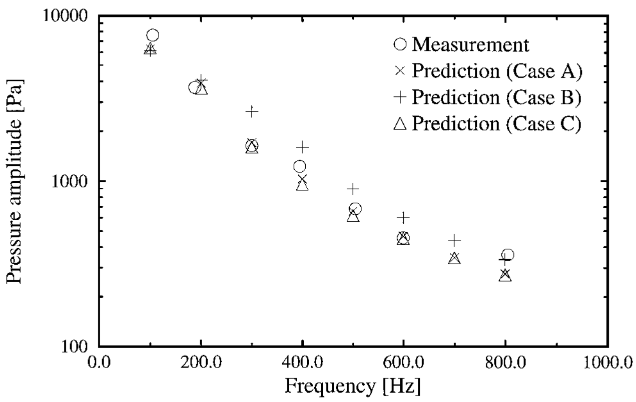
V. Results

This section discusses the results of the numerical simulations that investigated the performance of the controlled combustor. The first study predicted the open-loop response of the system and compared the results with experimental data.[11] The second numerical study investigated issues relevant to active control of combustion instabilities.

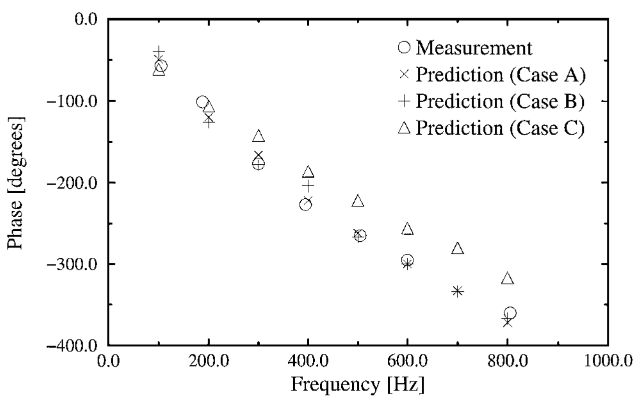
The initial conditions assume that only reactants are present in the space upstream of the secondary fuel injector location and that the remainder of the combustor is filled with products. Roe's scheme was employed to solve the overall continuity, momentum, and energy equations. The strengths of the characteristic waves used in Roe's solver were modified to account for the presence of heat addition.[21] The species conservation equations for the fuel and oxygen were separately solved using upwinding. The accuracy is of first-order in time and space. The secondary fuel injector is located close to the primary injector face ($x_s = 0.03$ m, see Fig. 1). Some conditions common to the two numerical examples are a Courant–Friedrichs–Lewy number of 0.7, an inlet temperature of 300 K for the reactants from the primary and secondary injectors, and a combustion chamber cross-sectional area of 1.14×10^{-3} m^2.

In example 1, the fuel flow rate supplied by the secondary (control) fuel injector into the combustor is pulsed sinusoidally at different frequencies. The response of the combustor is studied in terms of 1) the amplitude of the resultant pressure oscillations and 2) the phase between the secondary fuel injection and total heat-release oscillations. This comparison is a natural choice for the validation of our combustor model, which focuses on modeling combustion dynamics, because a dynamical system is often described in terms of gain and phase as a function of frequency. These open-loop studies were conducted in the parallel experimental work[11] to determine whether the actuator (i.e., the secondary fuel injector) can adequately influence conditions within the combustor and, thus, be suitable to serve as an actuator in closed-loop active control of combustion instabilities.

In the reported calculations, the mean chamber pressure is 3.56×10^5 Pa, the overall equivalence ratio is 0.85, the total mass flow rate is 1.28×10^{-2} kg/s, and the fraction of the total fuel injected through the secondary fuel injector is 0.18. These average values are held constant as the frequency of secondary fuel pulsation is varied. The amplitude of the secondary fuel flow rate oscillations is chosen to equal 100% of the average fuel flow through the secondary injector. The combustion chamber length equaled 0.24 m to ensure that its resonant frequencies are considerably higher than the frequencies at which the secondary fuel is pulsed. The computations were performed with 300 grid points, and the numerical accuracy was



a) Amplitude of pressure oscillation



b) Phase lag of the total heat release oscillations with respect to the secondary fuel oscillations

Fig. 3 Comparison of numerical predictions with data from experimental gaseous rocket motor for an open-loop test involving pulsation of the secondary fuel at different frequencies; model parameters: case A $\tau_{\text{mix}} = 1 \times 10^{-3}$ s and $\tau_{d,\text{mix}} = 1 \times 10^{-3}$ s, case B $\tau_{\text{mix}} = 8 \times 10^{-4}$ s and $\tau_{d,\text{mix}} = 1 \times 10^{-3}$ s, case C $\tau_{\text{mix}} = 1 \times 10^{-3}$ s and $\tau_{d,\text{mix}} = 8 \times 10^{-4}$ s and in cases A, B, and C $l_{\text{mix}} = 0.1$ m.

checked by doubling the number of grid points. Also, in agreement with the experimental study, the reactants' inflow into the combustor is assumed to be choked. Hence, the primary mass inflow [$\dot{m}_b$ in Eq. (7)] is a constant and, therefore, the time delay parameter $\tau_{d,c}$ in the injector boundary condition does not alter the imposed mass flux. The relevant model parameters are τ_{mix}, $\tau_{d,\text{mix}}$, and l_{mix}.

The model predictions are in good agreement with experimental data (see Fig. 3) when the mixing parameters τ_{mix}, $\tau_{d,\text{mix}}$, and l_{mix} equal 1×10^{-3} s, 1×10^{-3} s, and 0.1 m, respectively. The value for τ_{mix} was chosen based on the relevant timescales (10^{-4}–10^{-2} s) in turbulent combustion.[22] The experimental data for the phase shift shows a linear variation with frequency (see Fig. 3b) that is a characteristic of a pure time delay transfer function. Hence, $\tau_{d,\text{mix}}$, the pure time delay parameter in the heuristic model, was chosen such that the slope of the phase shift curve matched the corresponding value in the experimental data. The choice $l_{\text{mix}} = 0.1$ m may be interpreted as a rough estimate of the scale of the mixing region and was chosen based on radiation measurements in the experimental setup.[11]

For different values of the heuristic model parameters, curves qualitatively similar to those in Fig. 3 were obtained. Of the three model parameters τ_{mix}, $\tau_{d,\text{mix}}$, and l_{mix}, the heuristic model predictions could be varied in a wide range using τ_{mix} and $\tau_{d,\text{mix}}$ and, hence, the effect of varying these two parameters is discussed next.

The effect of the rate of mixing is studied by changing τ_{mix} from 1×10^{-3} to 8×10^{-4} s (compare cases A and B in Fig. 3). Note that this increase in the rate of mixing, obtained in the present model by choosing a smaller value for τ_{mix}, results in a less rapid decrease (compared to case A) of the pressure oscillation amplitude as the frequency is increased (see Fig. 3a). In the limit, if the heat release due to the secondary fuel injection occurs instantaneously after the secondary fuel is injected into the combustor, there would be no attenuation of the pressure oscillation amplitude as the frequency is increased. The change in τ_{mix} has no significant effect on the phase characteristics (see Fig. 3b).

In the experimental gas rocket,[11] the secondary fuel injector is located on the axis of the combustor and has orifices that are equally spaced in the azimuthal direction and oriented to radially inject the fuel toward the combustor wall. For a given amplitude of secondary fuel pulsation, it was possible to increase the amplitude of pressure oscillation to a certain extent by increasing the number of orifices in the secondary fuel injector and thereby increase mixing. This effect of change in the secondary fuel injector design on the pressure oscillation amplitude could be modeled by choosing a smaller value for τ_{mix}.

Comparison of cases A and C in Fig. 3 shows the effect of changing $\tau_{d,\text{mix}}$ from 1×10^{-3} to 8×10^{-4} s. The pressure oscillation amplitudes in cases A and C are approximately the same (see Fig. 3a). The predominant influence of a change in $\tau_{d,\text{mix}}$ is its effect on the slope of the phase shift variation with frequency (see Fig. 3b, cases A and C). Based on the analogy of the mixing source term with the transfer function $e^{-\tau_{d,\text{mix}} s}/(1 + \tau_{\text{mix}} s)$, one would expect that $\tau_{d,\text{mix}}$ would influence only the phase characteristics and would not have any effect on the pressure oscillation amplitude (because the magnitude of $e^{-\tau_{d,\text{mix}} s}$, after the substitution $s = j\omega$, is unity[23] for any frequency ω). However, there are differences between the heuristic model, which consists of nonlinear partial differential equations, and the transfer function analogy. For example, it was observed that larger variations in $\tau_{d,\text{mix}}$ with respect to case A resulted in significant variations in the pressure oscillation amplitudes.

Note that the response of the combustor to excitation, characterized by a decrease in pressure oscillation amplitude at higher frequencies, and a linear variation of phase with respect to frequency, has also been observed in studies of other types of combustors.[7–9] This example provides confidence that the heuristic model is capable of (at least) qualitatively capturing the overall trends in the dynamical behavior of the combustor.

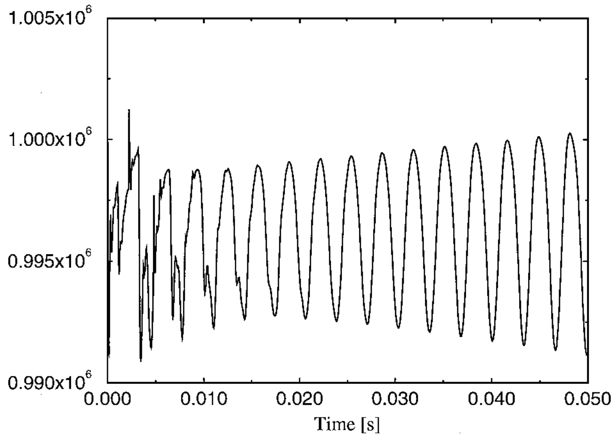
In example 2, we demonstrate a case of unstable combustion, its active control, and the effect of the amount of the secondary fuel pulsed on the performance of the active-control system. For this case, the primary injector is not choked and the mass inflow $\dot{m}_b$ of the primary reactants is obtained using the lumped parameter model for the feed system.

Depending on the choice of the model parameters, the combustor model exhibits stable or unstable combustion. In particular, an appropriate choice of $\tau_{d,c}$, the pure time delay imposed at the boundary, is useful for simulating large-amplitude instabilities. However, in this example we use only the values for the model parameters from example 1 (i.e., τ_{mix}, $\tau_{d,\text{mix}} = 1 \times 10^{-3}$ s each and $l_{\text{mix}} = 0.1$ m) and choose $\tau_{d,c} = 0.0$ s, thereby reducing the number of model parameters.

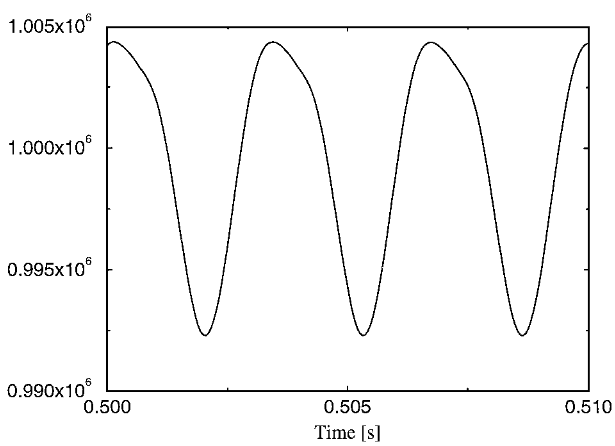
The mean chamber pressure is 1.0×10^6 Pa, the overall equivalence ratio is 0.7, the total mass flow rate is 1.0×10^{-2} kg/s, and the fraction of the total fuel injected through the secondary fuel injector is 0.2. The combustion chamber length is 1.57 m, and the computations are performed with 600 grid points.

The pressure oscillations at the injector face are shown as a function of time during the initial growth period (Fig. 4a), and after a limit cycle is attained (Fig. 4b). A Fourier transform of the limit cycle pressure oscillations (see Fig. 4c) shows that the instability consists of primarily the fundamental mode with a frequency of 307 Hz and also a small component at 614 Hz, the first harmonic.

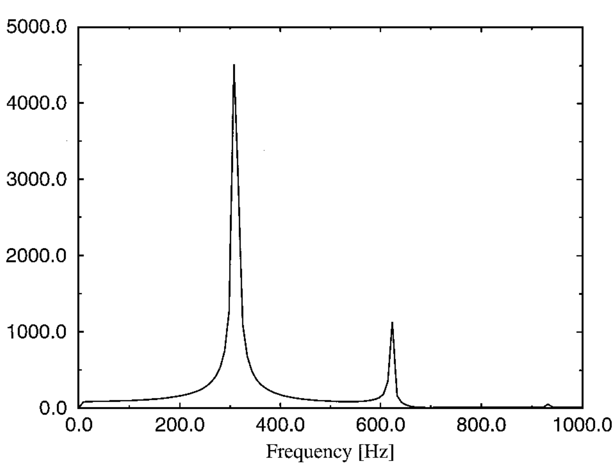
The pressure sensor is placed at the injector face because it is an antinode of all of the axial acoustic modes of the combustor. The pressure signal is sampled at 10 kHz and serves as the input to the observer. The control gain is chosen as 4.7×10^{-8} (kg/s)/Pa. The pure time delay τ_{con} discussed in Sec. IV is chosen as 6.0×10^{-4} s (six sampling times) based on an estimate from Nelson and Elliott.[19]



a) Pressure (injector face), Pa

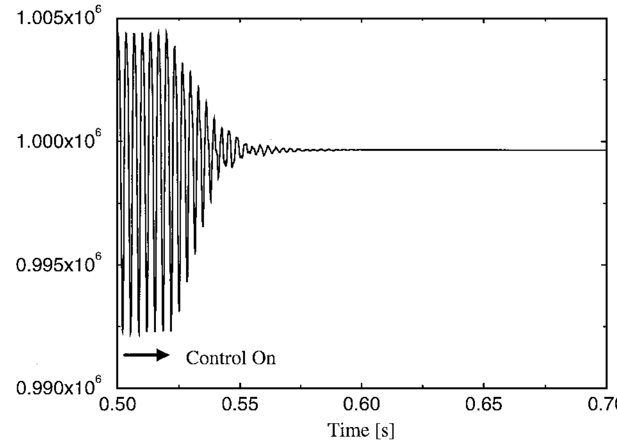


b) Pressure (injector face), Pa

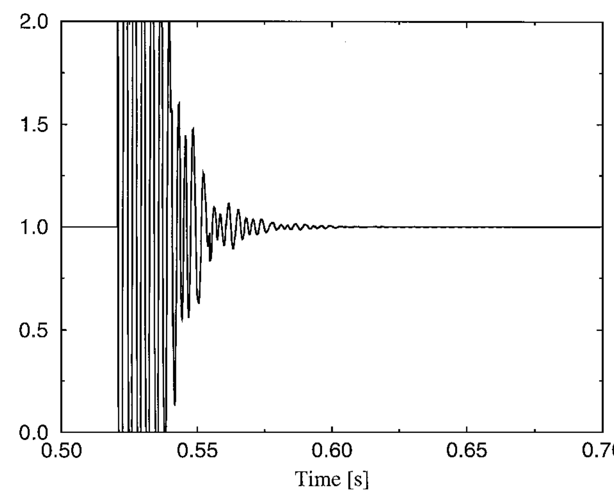


c) Fourier transform of injector face pressure

Fig. 4 Injector face pressure for an unstable mode of the combustor: a) initial growth of oscillations, b) oscillations after a limit cycle is attained, and c) Fourier transform of the pressure oscillations after limit cycle is attained.



a) Pressure (injector face), Pa



b) Secondary fuel injection (normalized)

Fig. 5 Active control of the instability shown in Fig. 4: a) injector face pressure oscillations and b) normalized secondary fuel injection.

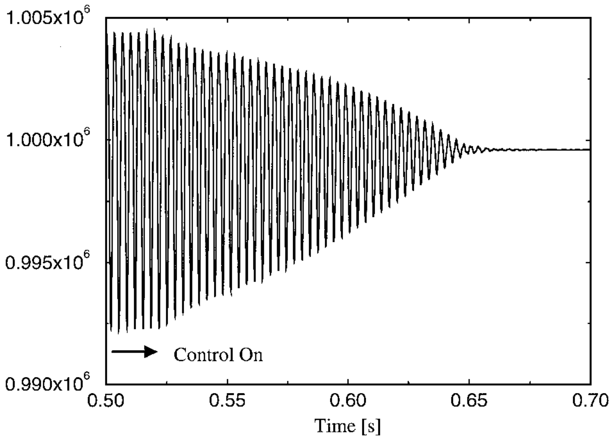
Consequently, in the numerical implementation, the output of the control system is delayed by a time interval of τ_{con} before being used to specify the amount of secondary fuel injection.

The control is turned on (i.e., the secondary fuel is allowed to be pulsed) at $t = 0.52$ s, after the instability has already attained a limit cycle. To demonstrate that the observer is capable of tracking the dominant modes in a short duration, the estimation of the nature of the instability by the observer is started at $t = 0.51$ s and the initial estimate of the observer frequency is chosen as 1 kHz. In this study, the observer is used to track only the most dominant mode of the instability.

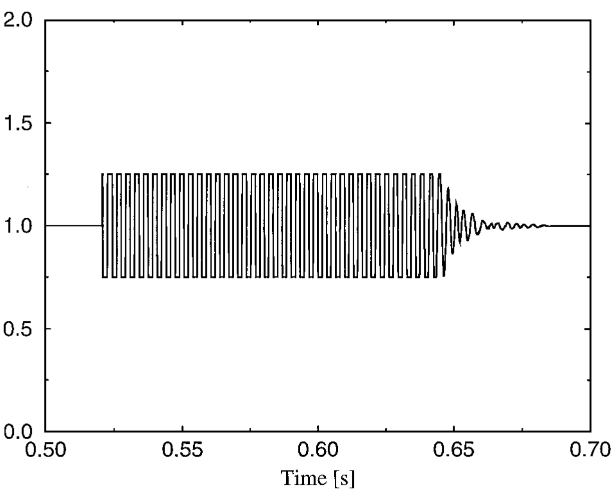
As shown in Fig. 5a, the instability is damped by the controller within a few cycles. The control effort expended is shown in Fig. 5b in terms of the normalized secondary fuel injection that is defined as the ratio of the amount of secondary fuel injected at a given time to the nominal amount of secondary fuel injection.

Next, the effect of the amount of secondary fuel pulsed on the control system performance is studied by limiting the amplitude of the secondary fuel pulsation to 25% of the secondary fuel (which corresponds to 5% of the total fuel flow); that is, the saturation limits for the normalized secondary fuel injection are 1 ± 0.25. The injector face pressure and the control effort expended for this case are shown in Figs. 6a and 6b, respectively. This provides an estimate for the amplitude of fuel pulsation required to obtain a specific rate of damping of the instability. When the amount of fuel pulsed was reduced further to 3.5% of the total fuel flow, the controller action did not completely damp the instabilities but only resulted in a 16% reduction in the amplitude of the limit cycle pressure oscillations with respect to the uncontrolled case.

Similar to this example, the combustor model can be used for studying different issues in active combustion control such as the effect of noise level on the active-control system performance, the control methodology, or the possibility of excitation of stable modes by the chosen active-control system.[24]



a) Pressure (injector face), Pa



b) Secondary fuel injection (normalized)

Fig. 6 Effect of amount of secondary fuel pulsed on control of instability (amplitude of secondary fuel pulsation limited to 25% of secondary fuel flow): a) injector face pressure oscillations and b) normalized secondary fuel injection.

VI. Conclusions

The main emphasis of this work is the development of a computationally inexpensive combustor model that involves only a few model parameters and that can simulate the general trends in the behavior of the combustor, particularly as a dynamical system. Good agreement between the trends in the model predictions and those in experimental data from a gas rocket, as well as other types of combustors, reveals the effectiveness of capturing the overall combustor behavior using the chosen model structure. Some model parameters also serve as controller parameters.

The structure of the model has been compared with other models such as IEM and a well-stirred reactor.

Active combustion instability control is demonstrated through an observer-based control system. The developed model serves as a platform to study different issues related to active combustion instability control.

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